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On the role of external reference frames on visual judgements of parallelity

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Abstract

In a previous study we found large systematic errors (up to 40°) when subjects adjusted the orientation of a horizontal test bar until it appeared parallel to a horizontal reference bar, both bars rotating about their vertical axes. The deviations increased linearly with the separation angle but vanished when the orientation of the reference bar was either parallel or perpendicular to the median line. In order to test the assumption that external references caused these deviations to vanish, the same task was repeated in four different conditions: in the normal condition the horizontal aperture, formed by a cabin, and the facing wall of the room were frontoparallel to the subject; in the other conditions either the room, the cabin or both were oriented 30° to the right with respect to the subject. It was found that, depending on the subject, the occurrence of the vanishing deviations covaried with the orientation of the cabin or the room. Evidently, subjects are influenced by the external references provided by the walls of the room and the sides of the cabin. The results indicate that a description of visual space by a Riemannian metric of constant curvature is not valid in a visual environment containing external references. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

It has long been known that many physical geometric relations in the environment appear distorted. For example, the perceived distance is compressed over a large range (Gilinsky, 1951), apparent parallel alleys and equidistance alleys are not physically parallel and equidistant (Blumenfeld, 1913; Indow & Watanabe, 1984), and apparent frontoparallel planes are not physically frontoparallel (Helmholtz, 1962; Ogle, 1964). These deviations from veridical have been confirmed by many authors for a broad range of tasks, conditions and settings (e.g. Battro, Reggini, & Karts, 1978; Foley, 1980; Heller, 1997; Higashiyama, 1984; Indow, 1997; Koenderink & Van Doorn, 1998; Wagner, 1985). However, what the relation actually is between the visually perceived space (in short visual space) and physical space, is still a matter of debate. In fact, there may not be a unique relation.

Luneburg (1947) introduced a Riemannian space of constant curvature as a description for visual space and proposed a model, which was developed further by Blank (1958, 1978), that could, at least qualitatively, describe all these departures from veridical (known at that time). However, a quantitative description remained cumbersome because the results were very task- and subject-dependent (Indow, 1991). Moreover, when the measurements were made outside and in broad daylight instead of in a dark room with faint luminous lights as stimuli, the Luneburg–Blank model was not appropriate (e.g. Battro, PierroNetto, & Rozestraten, 1976; Koenderink & Van Doorn, 1998; Koenderink, Van Doorn, & Lappin, 2000; Wagner, 1985): the results could be described more accurately with a simple distance function relating the perceived distance to the physical distance. Apparently, the context is an important factor for visual perception.

In a previous study (Cuijpers, Kappers, & Koenderink, 2000), we found large deviations from veridical (up to 40°) when subjects had to set a test bar parallel to a reference bar, both bars being arranged horizontally at eye height. The horizontal bars were viewed binocularly and could rotate about the vertical axes through their centres. The positions of the bars and the orientation of the reference bar were varied. The deviations were proportional to the separation angle between the test and reference bar. The mean proportion, averaged across the values found for the different orientations of the reference bar, ranged from 10% to 70% depending on the subject. Since the separation angle could be as large as 60°, a proportion of 70% would correspond to an average deviation of 42°. The deviations were independent of the distance of the bars from the observer. Surprisingly, for most subjects the deviations did not occur for two distinct orientations: for five out of eight subjects the deviation vanished when the reference bar was either parallel or perpendicular to the median line. It was suggested that this effect could have been due to external references, even though they had been minimised. In that particular experiment, the external references consisted of the walls of the room and a cabin with a horizontal aperture, in which the subject was seated. Both the cabin and the walls of the room were covered with wrinkled, black, plastic sheeting. The purpose of the plastic was to camouflage the external references and the purpose of the cabin was to prevent the subjects from seeing the floor and the ceiling.

Many studies in the literature are concerned with the role of the visual background on the visual perception of orientation (see also Howard, 1982). It has been shown that when subjects have to adjust the orientation of a rod to the true vertical, constant errors are made in the direction of rotation of a visual scene about the pitch axis (Asch & Witkin, 1948a) and about the roll axis (Asch & Witkin, 1948b; Goodenough, Cox, Sigman, & Strawderman, 1985; Howard & Childerson, 1994). In addition, subjects experienced a compelling illusion of self tilt opposite to the direction of rotation. Similarly, the orientation about the roll axis of a luminous rod in the dark is misjudged in the direction of rotation of a surrounding square luminous frame (Witkin & Asch, 1948). Goodenough, Oltman, Sigman, Rosso, and Mertz (1979) showed that the illusory self tilt could only partly explain these misjudgements. Ebenholtz and Utrie (1983) found that the effect of a rotated luminous frame may be cancelled by a circumscribing circular frame, but not by an inscribing frame. In all these studies a large between-subject variability was observed. According to Isableu, Ohlmann, Crémieux, and Amblard (1998) this is due to the different 'perceptive styles' subjects adopt to visually control their posture, i.e. whether dynamic or static visual cues are used. Clearly, the visual context is of major importance in judgements of orientation in the frontal and median plane. However, how orientations in the horizontal plane are judged when the visual scene is rotated about the vertical body axis (yaw) is not yet examined.

Several studies show that rotation of the visual context about the yaw axis affects visual (space) perception. For example, the apparent straight-ahead can slightly shift due to one or more flanking lights (Dietzel, 1924). Similarly, Roelofs (1935) discovered that the apparent direction of straight-ahead is displaced to the right if the left-hand edge of a luminous frame in the dark is placed in the objective median plane (Howard & Templeton, 1966). A visual scene rotating about the vertical body axis induces a compelling sensation of self rotation in the opposite direction (e.g. Howard & Howard, 1994). Schoumans, Koenderink, and Kappers (2000) showed that exocentric pointing is systematically affected by the yaw angle of the visual context, but the additional structure provided by the context did not lead to more consistent pointing.

The fact that the deviations from veridical disappear for non-oblique orientations of the reference bar superficially resembles the oblique effect reported in the literature (e.g. Appelle, 1972; Westheimer & Beard, 1998). However, in the literature the oblique effect occurred for orientations in the frontal plane instead of in the horizontal plane at eye height. To our knowledge the 'oblique effect' in the horizontal plane has not been reported before. The oblique effect in the frontal plane is typically explained by the fact that a disproportionate number of cells are optimally sensitive to stimuli in preferred orientations, and, in addition to this, by meridional anisotropies such as ocular astigmatism and the packing geometry of the retinal receptor mosaic. However, the 'oblique effect' of the parallelity task cannot originate from these anisotropies because all orientations in the horizontal plane project to only one orientation in the frontal plane, i.e. a horizontal line.

In the present study we test experimentally whether external references are the cause of the vanishing deviations for non-oblique orientations in the parallelity task.

For that purpose, we repeated our experiment for different orientations (about the vertical body axis) of the room and the cabin with respect to the subject. The separation between the test and reference bar and the orientation of the reference bar were varied whereas the distances of the bars from the subject were fixed at 1.47 m. With this setup we can determine not only whether the visual context has an effect on judgements of parallelity but also how a rotated visual background, formed by the walls of the room, and a rotated visual aperture, formed by the cabin, interact with these judgements. In order to address the role of the visual context completely it would be necessary to conduct this experiment in the dark with luminous bars as well. However, there is a strong indication that the 'oblique effect' of the parallelity task will disappear under these circumstances because subjects for whom no 'oblique effect' occurred show large deviations for every orientation of the reference bar (Cuijpers et al., 2000). For the present, we will assume that in a zero context the 'oblique effect' will indeed disappear.

Parallelity in Euclidean geometry has some special properties which are not true for more general geometries. For example, given a line and a point not on that line, then there exists a unique parallel line through that point. This line is parallel when both lines lie in a plane and do not intersect each other. However, in a spherical geometry such lines do not exist because they will always intersect each other and in a hyperbolical geometry there are infinitely many parallel lines. For these and more general geometries parallelity is defined locally: two vectors at the endpoints of a curve are said to be parallel in the sense of Levi-Civita (Stoker, 1969) when their orientations are the same after parallel-transport of one of the vectors along the curve. With parallel-transport is meant the displacement of a vector without changing its length and orientation along a given path. Although this definition of parallelity may look cumbersome, it is necessary because only in a flat geometry parallel-transport is independent of the path taken. In the current experiment subjects are asked to adjust the orientation of a test bar until it appears to have the same orientation as the reference bar. Consequently, this task closely resembles parallelity in the sense of Levi-Civita: the bars correspond to parallel vectors in visual space for some curve connecting the vectors. Depending on the structure of visual space some fundamental properties must hold. If visual space is Riemannian, then if the test bar looks parallel to the reference bar, the reference bar must also look parallel to the test bar. If visual space is a Riemannian space of constant curvature, then two bars which look parallel will also look parallel when rotated over the same physical angle. The first property was found to hold whereas the second property is violated by the oblique effect (Cuijpers et al., 2000).

2. Method

If it is true that external references provided by the cabin and the experimental room cause the deviations from veridical to vanish for two distinct orientations of the reference bar, then one would expect that for different orientations of the cabin and/or room the deviations would vanish for different orientations of the reference bar. For example, if the room and cabin are oriented α to the right relative to the subject and the stimulus positions, then one would expect the deviations to vanish for a reference orientation $-\alpha$ and $90^\circ - \alpha$ instead of the 0° and 90° orientation found previously. Since the reference orientations varied in steps of 30°, we chose $\alpha = 30^\circ$. In order to distinguish between the cabin and the room, four conditions were used in which the orientation of the room relative to the subject and the orientation of the cabin were varied. Note that the plastic sheeting used to camouflage these external references was still present.

2.1. Subjects

Four naive undergraduate students in their early twenties participated as subjects. All subjects had normal or corrected-to-normal vision: the visual acuity was at least visus 1 (tested with a Landoldt C letter chart) and stereo vision was better than 60 arcsec (tested with a standard TNO-test, Walraven, 1975). The measurements were conducted binocularly. The subjects received no feedback about their performance.

2.2. Experimental setup

The measurements took place in a 6 m × 6 m room with blinded windows and normal room-lighting conditions. The walls were covered with sheets of black plastic such that the corners were hidden and the background looked similar in all

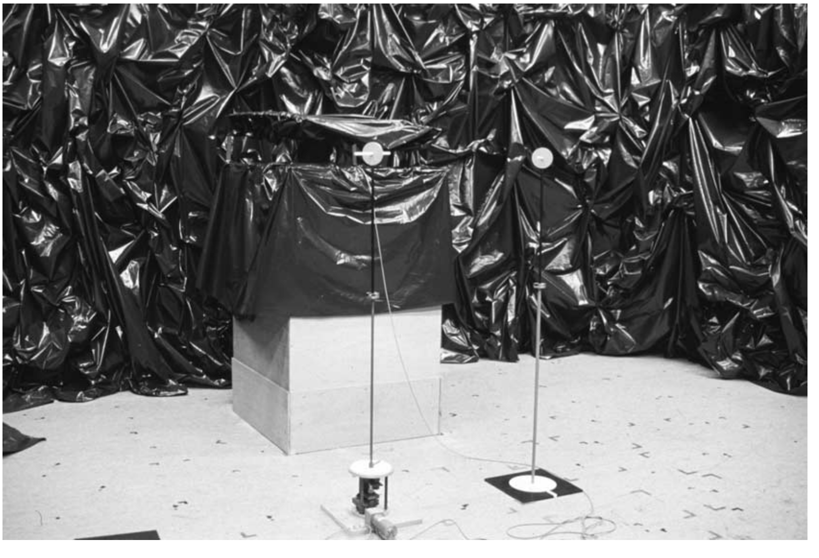

Fig. 1. Picture of the experiments room with the cabin. The walls are covered with black, wrinkled plastic. The subject is seated inside the cabin (height 150 cm, width 80 cm) the roof and sides of which prevent the subject from seeing the floor and the ceiling. In front of the cabin are the test (left) and reference bar (right) which are mounted on thin vertical, metal rods at a height of 1.38 m.

directions. The plastic was wrinkled in order to create a heavy ‘‘random’’ relief: the extent of the ridges and cavities was of the order of 10 cm (see Fig. 1).

The subject was seated on a chair which was adjustable in height and that was placed inside a small cabin (Fig. 1). The cabin consisted of three wooden side walls and a small roof. The back was open. Once seated in the cabin the subject could look through a horizontal aperture of 10 cm height between the sides and the roof. As a result, both the floor and the ceiling of the room were invisible. The visual field of the subjects extended about 10° vertically (see Fig. 2) and 210° degrees horizontally. The sides and the roof of the cabin were also covered with the black plastic.

The orientation of the subject was fixed by means of a chinrest mounted in the cabin. The relative orientation of the room and the cabin was varied with respect to the subject: in the normal condition (NC) the front of the cabin and the facing wall of the room were frontoparallel to the subject; in the second condition the cabin was rotated 30° to the right about the vertical axis through the chinrest (C30); in the third condition the room was oriented 30° to the right (R30) by physically rotating the subject (including the chair and the chinrest), the cabin and the stimuli 30° to the left about the vertical axis through the chinrest; in the fourth condition both the room and the cabin were oriented 30° to the right (RC30).

2.3. Stimuli

Two identical bars at a distance of 1.47 m were used as stimuli. Each bar consisted of a rod with pointed tips (top angle 60°) which protruded at right angles from each side of a circular disk (see Fig. 2). The length of the rod was 122 mm with a diameter of 5 mm and the diameter of the disk was 40 mm with a width of 5 mm. The rod was painted white and the disk yellow. The bars were mounted horizontally at eye height on thin vertical, metal rods and could be rotated in the horizontal plane (see Fig. 1). The thin vertical, metal rods also had a diameter of 5 mm and were painted black. This construction could be placed on another vertical, metal rod (not visible to the

Fig. 2. Picture from inside the cabin of the two bars that the subject perceived as parallel. The visual field extended about 10° vertically and 210° degrees horizontally.

subject) connected to a motor which the subject operated by remote control, or, alternatively, on a vertical, metal rod whose orientation the experimenter could adjust manually. The bar orientation could be read off from a scale that was invisible to the subject.

On the floor markers were used for the positioning of the bars. In condition NC the locations, expressed in polar co-ordinates, were (1.47 m, −30°), (1.47 m, 0°) and (1.47 m, 30°) with the 0°-direction along the axis of the room. The subjects were positioned such that the chinrest was directly above the origin and that the median line was in the 0°-direction (see Fig. 3). In conditions R30 and RC30 where the subject was rotated 30° to the left, the locations of the stimuli were adjusted accordingly to (1.47 m, 0°), (1.47 m, 30°) and (1.47 m, 60°).

2.4. Procedure

The subjects were asked to adjust a test bar, operated by remote control, such that it appeared parallel to a reference bar. The subjects were instructed with a drawing on a piece of paper so that it was clear that bars which are physically parallel have the same orientation in space. Before entering the room the subjects were asked to cover their eyes; when seated on the chair, they were allowed to see again. Consequently, the subjects could observe the room from a prescribed vantage point only. When the subjects entered the room, they were rotated about their vertical axis while being blindfolded so that they became disoriented. Their walking path was more or less arbitrary. While the test and reference bar were being positioned, the subjects

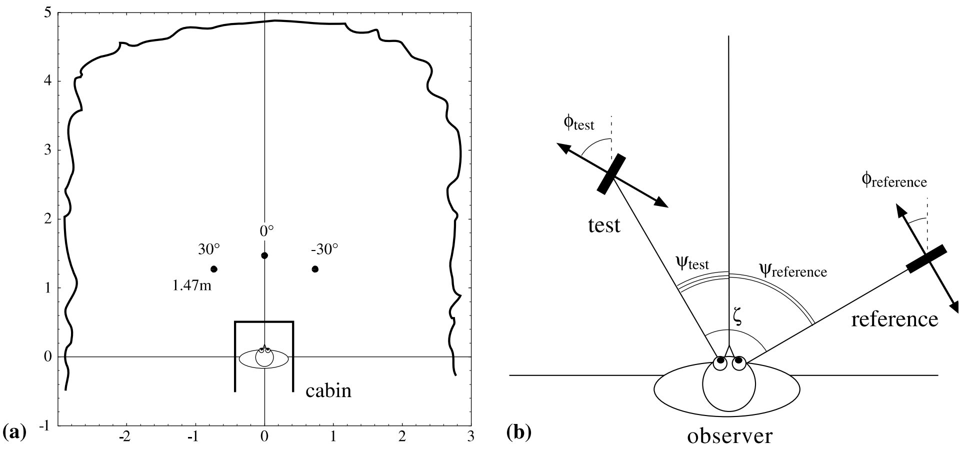

Fig. 3. (a) Schematic diagram of the top view of the room in the normal condition (NC). The bar positions used are indicated by the black dots. The subject is seated with the chin directly above the origin inside a cabin (indicated by the open square). The plastic sheeting covering the walls of the room is represented by the black curly line. Both the Cartesian (in m) and polar co-ordinates are indicated. (b) Diagram showing the definition of the orientation ϕ of both the test and reference bar. The relation between the separation angle ζ and the polar angles ψ of the bars is also shown.

had to keep their eyes closed. Once the bars were in position, the subjects were asked to rotate the test bar an arbitrary amount with their eyes still closed. In the meantime the orientation of the reference bar was being set by the experimenter. Upon a signal from the experimenter, the subjects opened their eyes and adjusted the orientation of the test bar until it looked parallel to the reference. The subjects signalled when they were satisfied and closed their eyes again. After that, the orientation was noted and the following trial was set up. No feedback was given about their performance.

The test and reference bar were always positioned at a distance of 1.47 m from the observer. The reference bar was placed at a polar angle of 30° or −30° (see Fig. 3(a)). The test bar was placed at 30°, 0° or −30° except at the reference position itself. For each configuration of the reference and test bar positions, six different reference orientations were measured, i.e. 0°, 30°, 60°, 90°, 120° and 150° in four conditions (NC, C30, R30, RC30). In all we performed 96 different trials. All trials were repeated three times and measured for four subjects. The total measuring time was about 20 h.

The trials were presented in four blocks, one for each condition. Each block was measured in a single session which usually took about 75 min to complete. For each block the order was randomised with respect to the test and reference position including the three repetitions. Three repetitions in a row of the same configuration were excluded. For each configuration all six orientations of the reference bar were presented in randomised order. The presentation order of the four blocks and the randomisation of the trials were different for each subject.

The orientation of the bar is expressed with the angle ϕ between the line through the bar and the 0°-line, which coincides with the axis of the room (see Fig. 3(b)). Instead of the actual orientation, we will be interested mainly in the difference between the orientation of the test and the reference bar which will be denoted by $\Delta\phi = \phi_{\text{test}} - \phi_{\text{reference}}$. A positive value corresponds to a counterclockwise deviation and vice versa. The position of each bar is expressed in its polar angles, i.e. ψ_{test} and $\psi_{\text{reference}}$, since the distance from the subject is always 1.47 m. In addition to the polar angles we will use the separation angle defined by $\zeta = \psi_{\text{test}} - \psi_{\text{reference}}$. Note that a negative value corresponds to the situation where the test bar is placed to the right of the reference bar.

3. Results

The analysis of the results is illustrated in detail for subject SB because for this subject the effects of the different conditions are the most pronounced. For the other subjects the intermediate steps are omitted and only the final results are shown.

In Fig. 4 the results are shown graphically and numerically for subject SB for a reference position of −30°. Each graph is a schematic drawing of a top view of the experimental room. The orientation is shown for the two positions of the test bar (thin lines) and the position of the reference bar (thick line). The indicated orientation of the test bar is the average of three repetitions. For subject SB the standard error of the mean is on average $2.0 \pm 0.2°$ which is similar to the other subjects

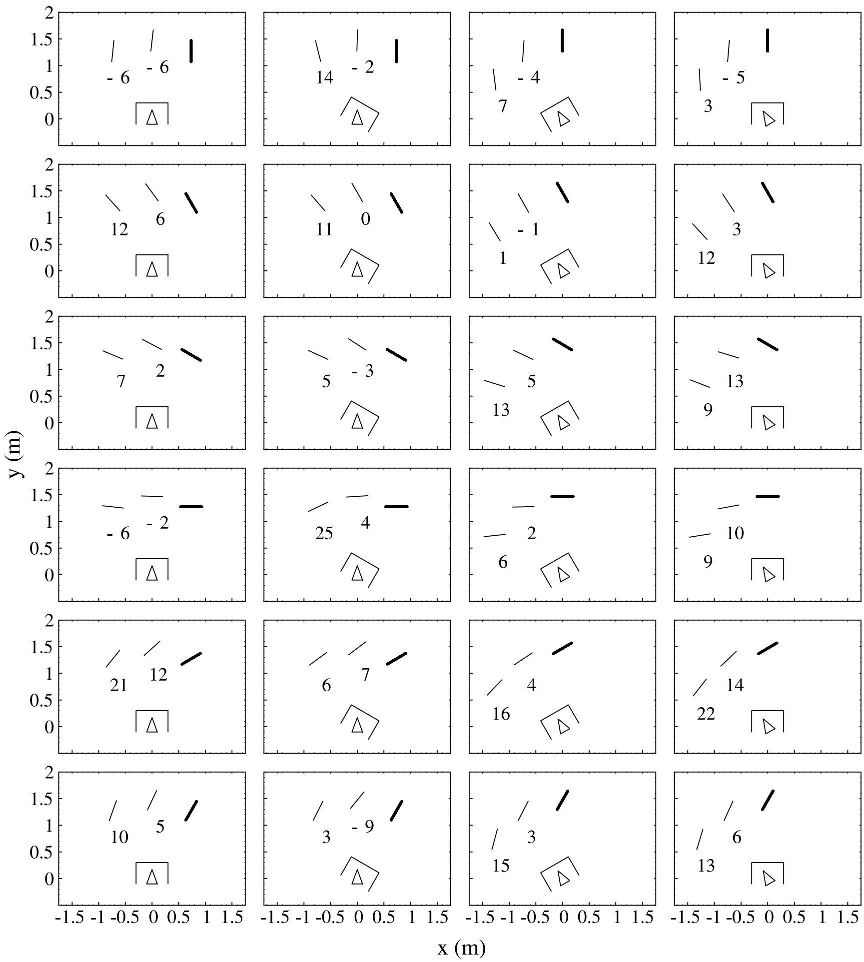

Fig. 4. Graphic and numerical representation of the settings of subject SB. The thick line corresponds to the position and orientation of the reference bar. The orientation of the test bar is indicated by a thin line for each position with the deviation in degrees shown directly below. The position and orientation of the cabin and the subject are represented by an open square and a triangle, respectively. From top to bottom the orientation of the reference bar varies from 0° to 150° in steps of 30°. From left to right the columns correspond to the conditions NC, C30, R30 and RC30.

(2.7 ± 0.2°, 2.3 ± 0.2° and 3.6 ± 0.3° for subjects KR, FL and DR, respectively). The deviations are indicated numerically directly below the test bar positions. The orientation and position of the subject and cabin are indicated by a triangle and a square with one open side, respectively. In each row a different orientation of the reference bar is shown, i.e. from top to bottom 0°, 30°, 60°, 90°, 120° and 150°. Note that the orientations are expressed with respect to the room and not with respect to the subject. The columns correspond to the different conditions: in consecutive order NC, C30, R30 and RC30.

In the first column of Fig. 4 it can be seen that in many situations there is a considerable positive deviation and that the deviations increase counterclockwise as the test bar is positioned further to the left of the reference bar. In other words, the deviation increases as the separation between the bars increases. However, for a reference orientation of 0° and 90° the deviations are negative and do not increase with increasing separation angle (first and fourth row, respectively). These results reproduce the results found previously (Cuijpers et al., 2000). In the second column the size of the deviations is very similar except that for a reference orientation of 0° and 90° large positive values occur and that the lowest values occur for 150° (which equals −30°) and 60°. In the third column the deviations are relatively small for 0°, 30° and 90°, and relatively large for 60°, 120° and 150°. In the last column the deviations are small for 0° only, although the deviations do not increase with the separation angle for the reference orientations of 60° and 90°.

For a reference position of 30° similar results are found with all deviations clockwise (not shown). These results can be combined with the previous results in a single figure by plotting the deviations from veridical ($\Delta\phi$) as a function of the separation angle ζ. This is illustrated for subject SB in Fig. 5 for each condition (columns) and reference orientation (rows). From top to bottom the reference orientations are 0°, 30°, 60°, 90°, 120° and 150°. From left to right the conditions are NC, C30, R30 and RC30. The diamonds correspond to single measurements and the line is a linear fit to the data. The regression coefficients are shown in the upper left corner. The positive and negative deviations correspond to a counterclockwise and clockwise deviation, respectively. The data for positive separation angles correspond to a reference position of −30°; these data were also shown in Fig. 4. The data for the negative separation angles correspond to a reference position of 30°.

From Fig. 5 it can be seen that, in good approximation, all the data are proportional to the separation angle. The offsets of the regression lines are in most cases negligible (not significant in 17 out of 24 cases at a confidence level of 95%). The vanishing deviations now appear as vanishing slopes which occur for different orientations of the reference bar for the different conditions: in condition NC (first column) the slope is not significantly different from zero for reference orientations of 0° and 90° ($t(10) < 1.25,\ P > 0.24$). For condition C30 (second column) the slope is negligible for 150° ($t(10) = 0.282,\ P = 0.783$) and significantly smaller than the average slope for 60° ($t(10) = 2.987,\ P = 0.007$). For condition R30 (third column) the slope is not significantly different from zero for 0° and 30° ($t(10) < 1.71,\ P > 0.11$). For condition RC30 (fourth column) the slope is not significant for 0° ($t(10) = 1.487,\ P = 0.165$).

Clearly, the slopes are very useful for summarising the effect of each condition on the settings of the subjects. Therefore the slopes are plotted as a function of the reference orientation ω_{ref} (with respect to the room). This is shown for all subjects in Fig. 6. The diamonds, squares, triangles and stars correspond to conditions NC, C30, R30 and RC30, respectively. The initials of the subjects and the standard error of the mean are indicated in the corners of each graph. For subject SB the slopes are significantly reduced or zero for the orientations mentioned earlier but are considerable for the other orientations, with values ranging from 0.11 to 0.28. For subject

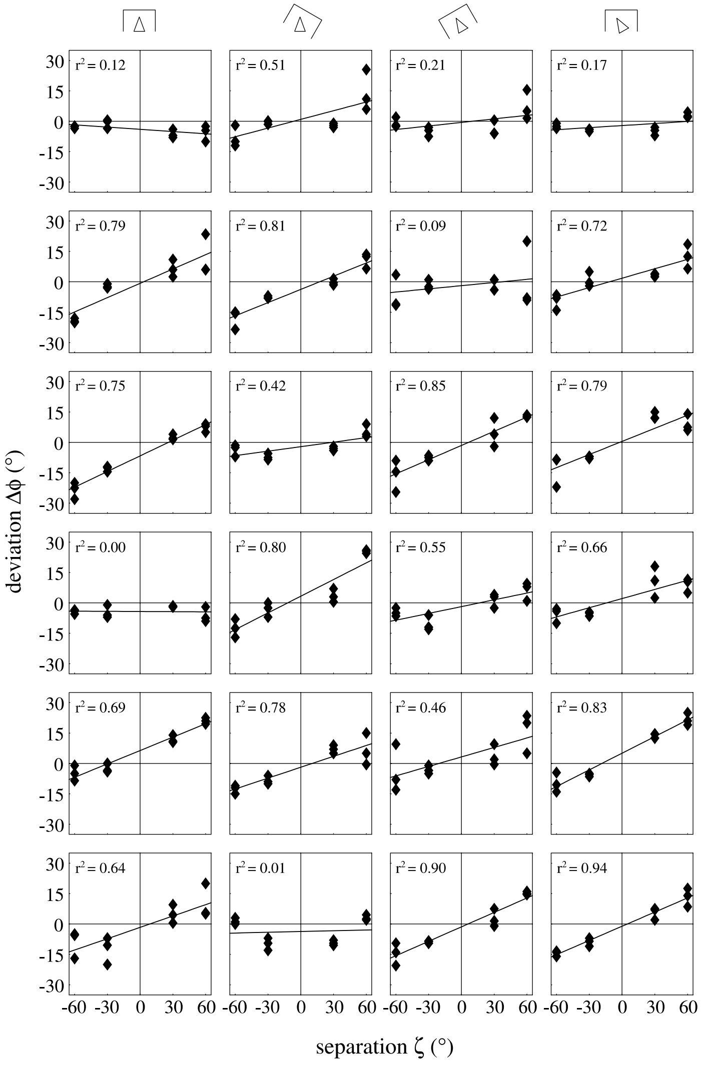

Fig. 5. The deviation $\Delta\phi$ from veridical as a function of the separation angle ζ for subject SB. From top to bottom the orientation of the reference bar varies from 0° to 150° in steps of 30°. From left to right the columns correspond to the conditions NC, C30, R30 and RC30. The squares of the regression coefficients are indicated in the corner of each graph.

KR the slopes are close to 0.5 in all conditions and for all orientations of the reference bar except when the reference orientation is 0° and, in condition R30 (triangles), for 0° and 30°. In these cases the slopes are on an average 0.14 and significantly smaller than the mean value of 0.40 ($t(10) < -2.27$, $P < 0.022$). For subject FL the slopes significantly reduce ($t(10) < -2.91$, $P < 0.007$) to about 0.12 for a reference orientation of 0° in conditions NC and RC30 (diamonds and stars).

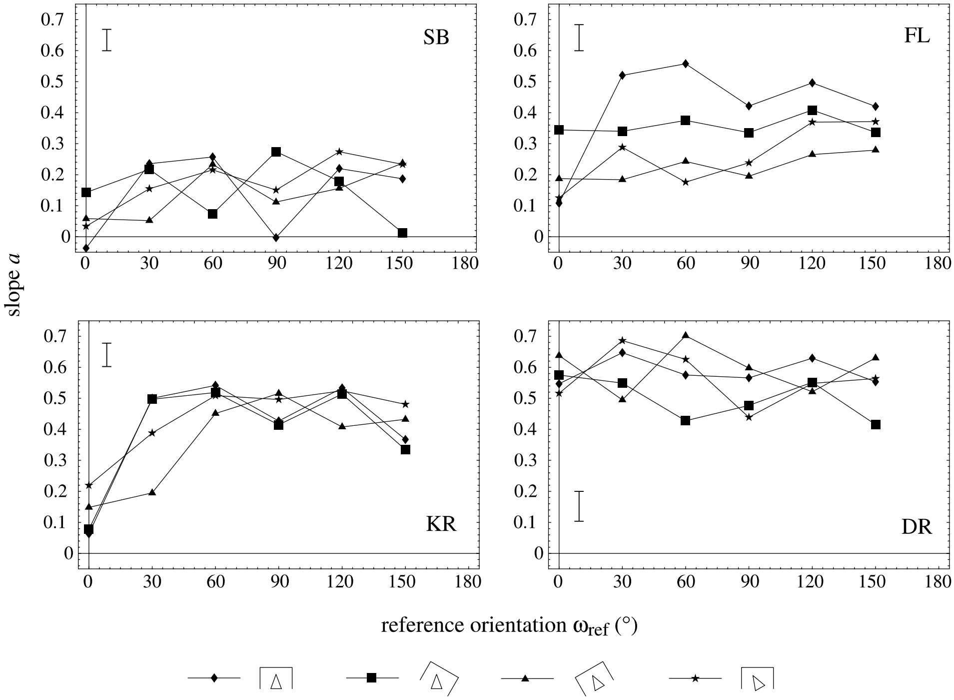

Fig. 6. Slopes of the fit $\Delta\phi = a\zeta$ as a function of the reference orientation (ω_{ref}) for each subject. The diamonds, squares, triangles and stars correspond to conditions NC, C30, R30 and RC30, respectively. The standard error of the mean and the initials of the subjects are shown in the corners of each graph.

Apart from these exceptions the slopes are in good approximation independent of the reference orientation. However, the average values vary from 0.2 to 0.5 for the different conditions. For subject DR the slopes are on an average 0.56 and approximately independent of both the reference orientation and the measuring condition. Only in 2 out 24 cases (for an orientation of 30° in condition R30 and for 90° in condition RC30) the slope was significantly smaller than the average slope of that condition ($t(10) < -2.06$, $P < 0.032$). The differences between the mean slopes of each condition and the total mean are negligible ($t(65) < 1.4$, $P > 0.16$). The average deviations vary considerably between subjects: the average slope for subject DR (0.56) is nearly four times as large as for subject SB (0.15).

4. Interpretation

Assuming that the slopes vanish due to some internal or external reference, one would expect the slopes to vanish for different orientations of the reference bar for each condition, depending on which reference is used. If an internal reference is used (or if the nose is used as a reference), one would expect the occurrence of the vanishing slopes to covary with the orientation of the subject. So if the slopes vanish for a 0° and 90 ° orientation of the reference bar in the normal condition (NC), one would expect the slopes to vanish for a 30° and 120° orientation of the reference bar in the conditions where the subject is rotated to the left with respect to the room (R30, RC30). In other words, one would expect the oblique effect to shift over +30°. Similarly, if the cabin acts as an external reference, one would expect a covariance with the orientation of the cabin and a shift of −30° when the cabin is oriented 30° to the right (C30) and a shift of +30° when it is oriented 30° to the left (R30). The same reasoning holds when the room acts as an external reference. In this case one would expect no shift of the oblique effect because the orientation of the reference bar is expressed with respect to the room. Note, however, that in this case the orientations do shift with respect to the subject. The expected shifts for each condition are illustrated in Fig. 7 for three hypotheses: the white columns represent the expected shifts when the oblique effect is related to the orientation of the subject, and the grey columns represent the expected shifts when the oblique effect is related to the orientation of the cabin. The third hypothesis is that the oblique effect is related to the orientation of the room, in which case no shifts are expected.

In order to address the shift of the oblique effect quantitatively, we calculate the discrete Fourier expansion of the data in Fig. 6 for each subject and condition and we analyse the phase of the lowest frequency components (see Appendix A for details). With this method we obtain a measure of the shift that is independent of the size of the oblique effect and, therefore, comparable between subjects. Moreover, noise due to the limited measuring accuracy can be effectively filtered out because it

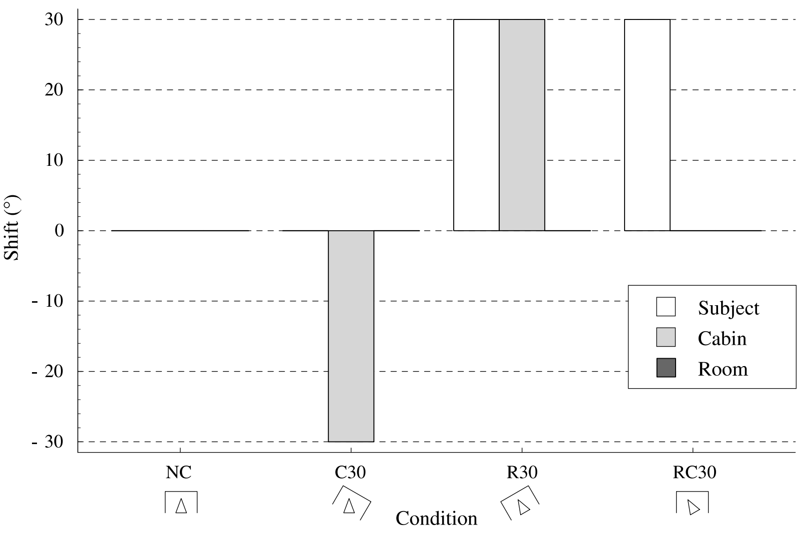

Fig. 7. Predicted shift of the oblique effect for each condition for three different hypotheses: the expected shift when the oblique effect is related to the orientation of the subject and the orientation of the cabin is indicated by the white and grey columns, respectively. When the oblique effect is related to the orientation of the room, the expected shift is zero for all conditions.

is present mainly in the high frequency components. The shift of the oblique effect occurs as a shift in the phase of each of the Fourier components. It turns out that, apart from the offset (zero frequency), only one component is large for all conditions and all subjects: this component is $-A\cos(4(\omega_{\mathrm{ref}} - \Delta\omega))$, which has a period of 90° and a shift of $\Delta\omega$. The values of the amplitude A and the shift $\Delta\omega$ depend on the subject and the condition. Note that the value of the shift $\Delta\omega$ does not have any meaning when the amplitude A is negligible. If a subject uses, for example, the cabin as an external reference, then the values of $\Delta\omega$ should equal the shift of the oblique effect and take the values 0°, −30°, 30° and 0° for conditions NC, C30, R30 and RC30, respectively (as is shown in Fig. 7).

The measured values of the shift $\Delta\omega$ are shown in Fig. 8 for each subject and condition. The error bars indicate the standard error of the mean estimated by regression of only the 90°-Fourier component. The conditions are shown on the x-axis and the different grey values of the columns correspond to the different subjects. The width of the columns corresponds to the amplitude A. A comparison of the measured shifts with the expected shifts reveals that subjects SB and DR behave very much as one would expect if the cabin is used as an external reference, whereas the measured shifts for subject KR and FL are most similar to what one would expect if the room was used as an external reference. However, the distinction is not very strict because for each subject there are exceptions to the rule: in the case of subject SB the measured shift in condition R30 is 13° instead of the expected 30° which is significantly different from either a 0° or 30° shift (the 95% confidence interval is (2.8°; 23.7°), $t(4) = 2.776$). This could be due to the fact that both the cabin and the room were used as an external reference. Alternatively, one could argue that both the room

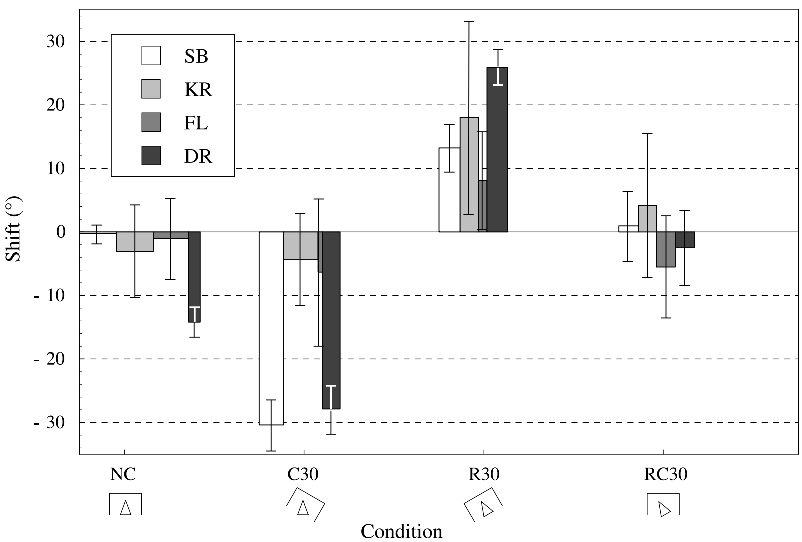

Fig. 8. Measured shift of the Fourier component with a 90° period for each subject and each condition. The error bars indicate the standard error of the mean.

and subject SB himself acted as a reference, but this does not seem likely because it is inconsistent with the observed shifts in conditions C30 and RC30. For subject DR a shift of -14° can be observed in condition NC. However, the corresponding amplitude (indicated by the width of the bars) is small ($A = 0.05$) compared to that for the other subjects (on average $A = 0.16$). This is due to the fact that subject DR does not have an oblique effect in condition NC (as was already shown in Fig. 6). For subject KR the measured shifts are consistent with the expected shifts when the room is used as a reference except for the observed shift of 18° in condition R30. However, the latter shift is not significantly different from either 0° or 30° (95% confidence interval is (−24°; 60°)). As in the case of subject SB, it is possible that both the cabin and the room were used as an external reference because a combination of a reference related to the subject and an external reference of the room is inconsistent with condition RC30. Finally, subject FL is most consistent with the hypothesis that the room is used as an external reference. The amplitude (or width of the bar) is very small in conditions C30 and R30 (0.02 and 0.03 compared to 0.16 and 0.08 in conditions NC and RC30, respectively), indicating that subject FL does not have an oblique effect for these conditions (see also Fig. 6). Since the orientations of the cabin and the room are not aligned in these conditions, it seems that subject FL, instead of using both the cabin and the room as a reference, ignored them both once their orientations were no longer the same. One could speculate that there might be a limit to how many external references can be used simultaneously.

5. Discussion and conclusions

For all subjects large deviations from veridical are found (up to 40°) which in good approximation are proportional to the separation angle. The slopes depend on the reference orientation and the measuring condition, except in the case of subject DR, for whom the slopes have approximately a constant value of 0.6. In the normal condition, which is the same condition as in our previous study (Cuijpers et al., 2000), the results are well reproduced: for three subjects (SB, KR and FL) the proportions are large for every reference orientation except for a reference orientation of 0°, where the deviations are significantly smaller or even zero. For subject SB this also occurs for a reference orientation of 90°. For subject DR the proportions are always large. In the other conditions a similar pattern was observed except that the reduced deviations shifted to other reference orientations (with respect to the subject) or even did not occur at all.

It was hypothesised that the walls of the room and the sides of the cabin act as an external reference enabling the subject to set the bars veridically parallel for the reference orientations of 0° and 90°. In that case, changing the orientation of the room and/or the cabin relative to the subject would change the reference orientations for which the deviations vanish. By analysing the shift in the oblique effect, we found that for subjects SB and DR the responses are most consistent with the hypothesis that the cabin is used as an external reference. On the other hand, the responses of subject KR and FL are most consistent with the hypothesis that the room is used as external reference. The distinction between whether the cabin or the room were used as an external reference is not very strict. However, the results indicate that, for all subjects, the oblique effect is not related to the orientation of the subjects themselves. Instead, it seems that subjects SB and KR used both the cabin and the room as an external reference in condition R30.

Evidently, subjects somehow use the external references provided by the room and the cabin. This happened despite the fact that the sides of the cabin and the walls of the room were camouflaged in order to conceal possible external references. It should be noted that the subjects are not consciously aware of the fact that the room and cabin can provide external references: in their opinion they attend only to the bars themselves and do not make large errors in setting the bars parallel.

If all external references were removed, one would expect the deviations not to vanish but to be large for all reference orientations. In that case, the deviations are independent of the reference orientation. The latter is a necessary condition for interpreting visual space as a Riemannian space of constant curvature (for details, see Cuijpers et al., 2000 and Stoker, 1969). Thus, the evidence presented here strongly suggests that a description of visual space as a constantly curved space is applicable only to a visual environment without external references.

Although the oblique effect may be removed by concealing all external references, the fact remains that large deviations are found. Hence it may be clear what causes the deviations to vanish, but it is unclear why these large deviations occur in the first place. By assuming that visual space has a metric, one could relate the deviations from veridical in the parallelity task to a distortion of the perceived distance. However, it remains to be seen whether a distorted distance perception is in fact the cause of the deviations that occur in this task.

In daily life our visual environment is crowded with possible external references. This could explain why the large deviations from veridical are not normally observed. On the other hand, it is unclear whether the visual system is sensitive to all external references at once or only one at a time. The fact that subject FL shows no dependence on the orientation of the reference bar when the cabin and the room do not have the same orientation could suggest that the context is ignored when the external references are ambiguous.

In conclusion, we find that the context provides external references which the visual system can use, even though the subjects are not consciously aware of using them. A description of visual space as a constantly curved Riemannian space is not valid unless all external references are removed.

Appendix A. Discrete fourier analysis

In our study, we found that the slopes are a function of the orientation of the reference bar. We wish to investigate the shift of this function for the different experimental conditions. Since the shape of the function differs across conditions and subjects, we use a discrete Fourier expansion to analyse the data. In the ideal case, each frequency component would have the same shift and the phase would be equal to the product of the frequency and the shift. In practice, the phase can only be determined reliably from the frequency components with a large amplitude. Furthermore, large shifts can only be obtained from the lower frequency components because of the periodic nature of the phase of each component. This is fortunate, however, because noise will be present mainly in the higher frequency components due to its random nature. Therefore, we analyse the phase of a low frequency component with a sufficiently large amplitude.

In general, the discrete Fourier series $\widehat{f}$ of a function with values f_k with $k = 0, \ldots, n-1$ is given by

$$\widehat{f}_k = \sum_{j=0}^{n-1} A_j \cos\left(\frac{2\pi k j}{n} - \varphi_j\right), \tag{A.1}$$

where $A_j = |a_j|$ and $\varphi_j = \arg(a_j)$ are the amplitude and the phase of the jth Fourier component, respectively. The values of the amplitude and the phase are determined by taking, respectively, the modulus and the argument of the complex Fourier transform a_j which is defined by

$$a_j = \frac{1}{n} \sum_{k=0}^{n-1} f_k \exp\left(\frac{2\pi i}{n} k j\right). \tag{A.2}$$

In our case, the function values f_k are given by the slope values for the discrete orientations ω_k of the reference bar. The orientation of the reference bar was varied in steps of 30°, so we have $\omega_k = (\pi/6)k$ with $k = 0, \ldots, n-1$ and $n = 6$. In the normal condition, the deviation vanished for a reference orientation of 0°. As a consequence, the main Fourier component will have a minimum for 0° and the corresponding phase φ_j will be equal to π. By using the identity $\cos(x - \varphi) = -\cos(x - (\varphi - \pi))$, we obtain the shift with respect to the normal condition

$$\widehat{f}(\omega_k) = A_0 - \sum_{j=1}^{5} A_j \cos(2j(\omega k - \Delta\omega_j)), \tag{A.3}$$

where $\Delta\omega_j = (\varphi_j - \pi)/2j$ is the shift we wish to determine. Thus, the shift for each frequency component is given by

$$\Delta\omega_j = \frac{1}{2j}(\arg(a_j) - \pi). \tag{A.4}$$

An important consequence of the discrete character of the Fourier expansion is that not all components are independent: the contribution of the jth component to f_k in Eq. (A.1) is identical to the contribution of the $(n - j + 1)$th component (except when $j = 0$ and, if n is even, when $j = n/2$). As a result, only the first four components of Eq. (A.3) are independent. The fifth and sixth component can be accounted for by doubling the amplitude of the third and second component, respectively. Thus, the Fourier expansion becomes

$$\widehat{f}(\omega_k) = A_0 - 2\sum_{j=1}^{2} A_j \cos(2j(\omega_k - \Delta\omega_j)) - A_3 \cos(6(\omega_k - \Delta\omega_3)). \quad \text{(A.5)}$$

A.1. Example

In the normal condition the oblique effect occurs at a reference orientation of 0° for most subjects. The form of Eq. (A.3) is chosen such that $\widehat{f}$ has a minimum for 0° when all $\Delta\omega_j$ are zero. Hence, any shift in the oblique effect for the other conditions is obtained with respect to the normal condition. As an example, we use the data for subject SB (see Fig. 6) in the case where the cabin is oriented 30° to the right with

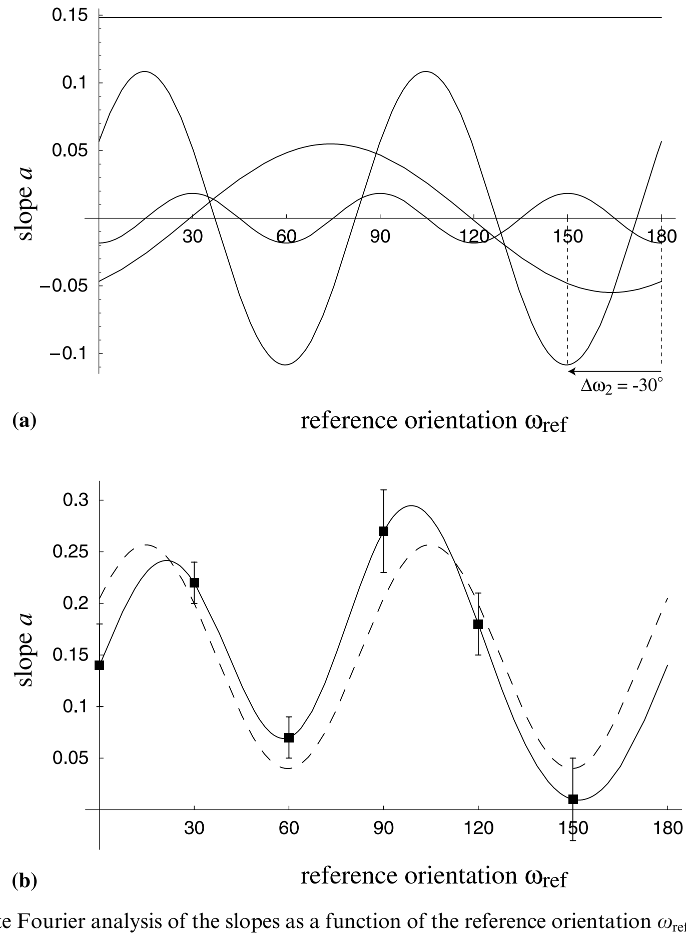

Fig. 9. Discrete Fourier analysis of the slopes as a function of the reference orientation ω_{ref}: (a) Individual Fourier components. The shift of the Fourier component with the largest amplitude (apart from the offset) is indicated. (b) The original data (indicated by the boxes), the sum of all Fourier components (solid line) and the sum of the offset and the largest Fourier component (dashed line). The error bars indicate the standard error of the mean.

respect to the subject. In Fig. 9(a) each Fourier component, calculated from Eq. (A.5), is drawn separately. As can be seen, the offset has the largest amplitude ($A_0 = 0.15$), but, obviously, no shift can be obtained from the offset. The second largest amplitude is obtained for the second Fourier component ($A_2 = 0.11$). For this component the minimum values occur at approximately 60° and 150°, which corresponds to a shift of $\Delta\omega_2 = -30°$. Similarly, the shift can be determined for the other Fourier components, yielding $\Delta\omega_1 = -16°$ and $\Delta\omega_3 = 0°$. However, because of the small amplitudes, these components hardly affect the location of the minima. Most of the modulation corresponding to the oblique effect is determined by the second Fourier component. This is illustrated in Fig. 9(b): in addition to the original data (indicated by the boxes) two curves are shown. The solid curve represents the sum of all Fourier components. The dashed curve represents the sum of only the offset (A_0) and the second Fourier component ($j = 2$). It can be seen that the locations of the minima of the dashed curve are in close agreement with those of the solid curve. Furthermore, the 95% confidence intervals of the data (corresponding to the error bars multiplied by $t_{0.975}(10) = 2.28$) overlap with the dashed curve. Thus, the second Fourier component provides a good description of the oblique effect and the corresponding shift $\Delta\omega_2$ is a good quantitative measure of the shift of the oblique effect.

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